

A 3+1 Dimensional Light-Front Model with Spontaneous Breaking of Chiral Symmetry

M. Burkardt and H. El-Khozondar

*Department of Physics
New Mexico State University
Las Cruces, NM 88003-0001
U.S.A.*

We investigate a 3+1 dimensional toy model that exhibits spontaneous breakdown of chiral symmetry, both in a light-front (LF) Hamiltonian and in a Euclidean Schwinger-Dyson (SD) formulation. We show that both formulations are completely equivalent — provided the renormalization is properly done. For the model considered, this means that if one uses the same transverse momentum cutoff on the SD and LF formulations then the vertex mass in the LF calculation must be taken to be the same as the current quark mass in the SD calculation. The kinetic mass term in the LF calculation is renormalized non-trivially, which is eventually responsible for the mass generation of the physical fermion of the model.

I. INTRODUCTION

Light-front (LF) coordinates are natural coordinates for describing scattering processes that involve large momentum transfers — particularly deep inelastic scattering. This is because correlation functions at high energies are often dominated by the free quark singularities which are along light-like direction. This is one of the main motivations for formulating field theories in the LF framework [1]. But light-front field theories have another peculiar feature, namely naive reasoning suggests that the vacuum of all LF Hamiltonians is equal to the Fock vacuum [2,1]. It thus *appears* as if LF Hamiltonians (at least those without the so-called zero-modes, i.e. modes with $k^+ = 0$) cannot be able to describe theories where the vacuum has any nontrivial features, such as QCD — where chiral symmetry is believed to be spontaneously broken. Even if one is only interested in parton distributions, one might be worried about using a framework where the vacuum is just empty space to describe a theory like QCD.

However, it is not quite so easy to dismiss LF field theory as the following few examples show: One of the first field theories that was completely solved in the LF formulation was QCD_{1+1} ($N_C \rightarrow \infty$) [3]. 't Hooft's solution did not include any zero-modes and therefore he had a trivial vacuum. Nevertheless, his spectrum agreed perfectly well with the numerical results from calculations based on equal time Hamiltonians [4]. Beyond that, application of current algebra sum rules to spectrum and decay constants obtained from the LF calculation formally gave nonzero values for the quark condensates that also agreed with numerical results at equal time [5]. This peculiar

result could be understood by defining LF field theory through a limiting procedure, which showed that some observables (here: spectrum and decay constants) have a smooth and continuous LF limit, while others (here quark condensates) have a discontinuous LF limit. Other examples have been studied, in which it was still possible to demonstrate equivalence between LF results and equal time results nonperturbatively, provided the LF Hamiltonian was appropriately renormalized [7–9]. Even though these examples are just 1+1 dimensional field theories, it is generally believed among the optimists in the field [10,11,1] that it should be possible in 3+1 dimensional field theories as well to achieve equivalence between spectra of LF Hamiltonian and equal time Hamiltonians by appropriate renormalization. However, no nontrivial examples (examples that go beyond mean field calculations) to support such a belief existed so far.

In this paper, we will give a 3+1 dimensional toy model that can be solved both in a conventional framework (here by solving the Schwinger-Dyson equations) as well as in the LF framework. We will unashamedly omit zero modes as explicit degrees of freedom throughout the calculation. Nevertheless, we are able to show that appropriate counter-terms to the LF Hamiltonian are sufficient to demonstrate equivalence of the spectrum and other physical properties between the two frameworks.

II. A SIMPLE TOY MODEL

The model that we are going to investigate consists of fermions with some “color” degrees of freedom (fundamental representation) coupled to the transverse component of a vector field, which also carries “color” (adjoint representation). The vector field does not self-interact.¹ Furthermore, we will focus on the limit of an infinite number of colors: $N_C \rightarrow \infty$ (g fixed), which will render the model solvable in the Schwinger-Dyson approach

¹Note that, for finite N_C , box diagrams with fermions would induce four boson counter-terms, which we will ignore here since we will consider the model only in the large N_C limit.

$$\mathcal{L} = \bar{\psi} \left(i\not{\partial} - m - \frac{g}{\sqrt{N_C}} \vec{\gamma}_\perp \vec{A}_\perp \right) \psi - \frac{1}{2} \text{tr} \left(\vec{A}_\perp \square \vec{A}_\perp + \lambda^2 \vec{A}_\perp^2 \right). \quad (2.1)$$

With “ \perp component” we mean here the x and y components. Furthermore we will impose a transverse momentum cutoff on the fields and we will consider the model at fixed cutoff. Also, even though we are interested in the chiral limit of this model, we will keep a finite quark mass since the LF formulation has notorious difficulties in the strict $m = 0$ case. Those difficulties can be avoided if one takes $m > 0$ considers $m \rightarrow 0$.

Even though certain elements of the model bear some resemblance to terms that appear in the QCD Lagrangian, the model seems is a rather bizarre construction. However, there is a reason for this: What we are interested in is a LF-investigating of a model that exhibits spontaneous breakdown of chiral symmetry. Furthermore, we wanted to be able to perform a “reference calculation” in a conventional (non-LF) framework. In the large N_C limit, the rainbow approximation for the fermion self-energy becomes exact, which allows us to solve the model exactly in the Schwinger-Dyson approach. The vector coupling of the bosons to the fermions was chosen because it is chirally invariant and because a similar coupling occurs in regular QCD. The restriction to the \perp component of the fields avoids interactions involving “bad currents”. Finally, using a transverse momentum cutoff both in the Schwinger-Dyson approach and in the LF calculation should allow us to directly compare the two formulations.

A. Schwinger-Dyson Solution

Because the above toy model lacks full covariance (there is no symmetry relating longitudinal and transverse coordinates) the full fermion propagator is of the form

$$S_F(p^\mu) = \not{p}_L S_L(\vec{p}_L^2, \vec{p}_\perp^2) + \not{p}_\perp S_\perp(\vec{p}_L^2, \vec{p}_\perp^2) + S_0(\vec{p}_L^2, \vec{p}_\perp^2), \quad (2.2)$$

where $\not{p}_L \equiv k_0 \gamma^0 + k_3 \gamma^3$ and $\not{p}_\perp \equiv k_1 \gamma^1 + k_2 \gamma^2$. On very general grounds, it should always be possible to write down a spectral representation for S_F^2

$$S_i(\vec{p}_L^2, \vec{p}_\perp^2) = \int_0^\infty dM^2 \frac{\rho_i(M^2, \vec{p}_\perp^2)}{\vec{p}_L^2 - M^2 + i\varepsilon}, \quad (2.3)$$

where $i = L, \perp, 0$. Note that this spectral representation differs from what one usually writes down as a spectral representation in that we are not assuming full covariance here. Note that in a covariant theory, one usually writes down spectral representations in a different form, namely $S = \int_0^\infty d\tilde{M}^2 \tilde{\rho}(\tilde{M}^2) / (\vec{p}_L^2 - \vec{p}_\perp^2 - \tilde{M}^2)$, i.e. with \vec{p}_\perp^2 in the denominator. This is a special case of Eq. (2.3) with $\rho(M^2, \vec{p}_\perp^2) = \int_0^\infty d\tilde{M}^2 \tilde{\rho}(\tilde{M}^2) \delta(M^2 - \tilde{M}^2 - \vec{p}_\perp^2)$.

Using these above ansatz (2.3) for the spectral densities, one finds for the self-energy

$$\begin{aligned} \Sigma(p^\mu) &\equiv ig^2 \int \frac{d^4 k}{(2\pi)^4} \vec{\gamma}_\perp S_F(p^\mu - k^\mu) \vec{\gamma}_\perp \frac{1}{k^2 - \lambda^2 + i\varepsilon} \\ &= \not{p}_L \Sigma_L(\vec{p}_L^2, \vec{p}_\perp^2) + \Sigma_0(\vec{p}_L^2, \vec{p}_\perp^2), \end{aligned} \quad (2.4)$$

where

$$\begin{aligned} \Sigma_L(\vec{p}_L^2, \vec{p}_\perp^2) &= g^2 \int_0^\infty dM^2 \int_0^1 dx \int \frac{d^2 k_\perp}{8\pi^3} \frac{(1-x) \rho_L(M^2, (\vec{p} - \vec{k})_\perp^2)}{D} \\ \Sigma_0(\vec{p}_L^2, \vec{p}_\perp^2) &= -g^2 \int_0^\infty dM^2 \int_0^1 dx \int \frac{d^2 k_\perp}{8\pi^3} \frac{\rho_0(M^2, (\vec{p} - \vec{k})_\perp^2)}{D}. \end{aligned} \quad (2.5)$$

and

$$D = x(1-x)\vec{p}_L^2 - xM^2 - (1-x)(\vec{k}_\perp^2 + \lambda^2) \quad (2.6)$$

Note that Σ_\perp vanishes, since $\sum_{i=1,2} \gamma_i \gamma_j \gamma_i = 0$ for $j = 1, 2$. Self-consistency then requires that

$$S_F = \frac{1}{\not{p}_L [1 - \Sigma_L(\vec{p}_L^2, \vec{p}_\perp^2)] + \not{p}_\perp - [m + \Sigma_0(\vec{p}_L^2, \vec{p}_\perp^2)]} \quad (2.7)$$

In the above equations we have been sloppy about cut-offs in order to keep the equations simple, but this can be easily remedied by multiplying each integral by a cutoff on the fermion momentum, such as $\Theta(\Lambda_\perp^2 - (\vec{p} - \vec{k})_\perp^2)$. In principle, the set of equations [Eqs. (2.4), (2.5), (2.7)] can now be used to determine the spectrum of the model. But we are not going to do this here since we are more interested in the LF solution to the model. However, we would still like to point out that, for large enough g , one obtains a self-consistent numerical solution to the Euclidean version of the model which has a non-vanishing scalar piece — even for vanishing current quark mass m , i.e. chiral symmetry is spontaneously broken and a dynamical mass is generated for the fermion in this model.

B. LF Solution

A typical framework that people use when solving LF quantized field theories is discrete light-cone quantization

²What we need is that the Green's functions are analytic except for poles and that the location of the poles are consistent with longitudinal boost invariance (which is manifest in our model). The fact that the model is not invariant under transformations which mix p_L and p_\perp does not prevent us from writing down a spectral representation for the dependence on p_L .

(DLCQ) [13]. Since it is hard to take full advantage of the large N_C limit in DLCQ, we prefer to use a Green's function framework based on a 4 component formulation of the model.

In a LF formulation of the model, the fermion propagator (to distinguish the notation from the one above, we denote the fermion propagator by G here) should be of the form³

$$G(p^\mu) = \gamma^+ p^- G_+(2p^+ p^-, \vec{p}_\perp^2) + \gamma^- p^+ G_-(2p^+ p^-, \vec{p}_\perp^2) + \not{k}_\perp G_\perp(2p^+ p^-, \vec{p}_\perp^2) + G_0(2p^+ p^-, \vec{p}_\perp^2). \quad (2.8)$$

Again we can write down spectral representations

$$G_i(2p^+ p^-, \vec{p}_\perp^2) = \int_0^\infty dM^2 \frac{\rho_i^{LF}(M^2, \vec{p}_\perp^2)}{2p^+ p^- - M^2 + i\varepsilon}, \quad (2.9)$$

where $i = +, -, \perp, 0$. This requires some explanation: On the LF, one might be tempted to allow for two terms in the spectral decomposition of the term proportional to γ^+ , namely

$$tr(\gamma^- G) \propto \int_0^\infty dM^2 \frac{p^- \rho_a(M^2, \vec{p}_\perp^2) + \frac{1}{p^+} \rho_b(M^2, \vec{p}_\perp^2)}{2p^+ p^- - M^2 + i\varepsilon}. \quad (2.10)$$

However, upon writing

$$\frac{1}{p^+} = \frac{1}{p^+ M^2} (M^2 - 2p^+ p^-) + \frac{2p^-}{M^2} \quad (2.11)$$

one can cast Eq. (2.10) into the form

$$tr(\gamma^- G) \propto \int_0^\infty dM^2 p^- \frac{\rho_a(M^2, \vec{p}_\perp^2) + \frac{2}{M^2} \rho_b(M^2, \vec{p}_\perp^2)}{2p^+ p^- - M^2 + i\varepsilon} - \frac{1}{p^+} \int_0^\infty dM^2 \frac{\rho_b(M^2, \vec{p}_\perp^2)}{M^2}, \quad (2.12)$$

which is of the form in Eq.(2.9) plus an energy independent term. The presence of such an additional energy independent term would spoil the high energy behavior of the model [12]: In a LF Hamiltonian, not all coupling constants are arbitrary. In many examples, 3-point couplings and the 4-point couplings must be related to one another so that the high energy behavior of scattering via the 4-point interaction and via the iterated 3-point interaction cancel [12]. If one does not guarantee such a cancellation then the high-energy behavior of the LF formulation differs from the high-energy behavior in covariant field theory and in addition one often also gets a spectrum that is unbounded from below. In Eq. (2.12), the energy independent constant appears if the coupling

constants of the "instantaneous fermion exchange" interaction in the LF Hamiltonian and the boson-fermion vertex are not properly balanced. In the following we will assume that one has started with an ansatz for the LF Hamiltonian with the proper high-energy behavior, i.e. we will assume that there is no such energy independent piece in Eq. (2.12).

The LF analog of the self-energy equation is obtained by starting from an expression similar to Eq.(2.5) and integrating over k^- . One obtains

$$\Sigma^{LF} = \gamma^+ \Sigma_+^{LF} + \gamma^- \Sigma_-^{LF} + \Sigma_0^{LF}, \quad (2.13)$$

where

$$\begin{aligned} \Sigma_+^{LF}(p) &= g^2 \int_0^\infty dM^2 \int_0^{p^+} dk^+ \int \frac{d^2 k_\perp}{16\pi^3} \frac{\left(p^- - \frac{\lambda^2 + \vec{k}_\perp^2}{2k^+}\right) \rho_+^{LF}(M^2, (\vec{p} - \vec{k})_\perp^2)}{k^+ (p^+ - k^+) D^{LF}} \\ &\quad + CT \\ \Sigma_-^{LF}(p) &= g^2 \int_0^\infty dM^2 \int_0^{p^+} dk^+ \int \frac{d^2 k_\perp}{16\pi^3} \frac{(p^+ - k^+) \rho_-^{LF}(M^2, (\vec{p} - \vec{k})_\perp^2)}{k^+ (p^+ - k^+) D^{LF}} \\ \Sigma_0^{LF}(p) &= -g^2 \int_0^\infty dM^2 \int_0^{p^+} dk^+ \int \frac{d^2 k_\perp}{16\pi^3} \frac{\rho_0^{LF}(M^2, (\vec{p} - \vec{k})_\perp^2)}{k^+ (p^+ - k^+) D^{LF}}. \end{aligned} \quad (2.14)$$

where

$$D^{LF} = p^- - \frac{M^2}{2(p^+ - k^+)} - \frac{\lambda^2 + \vec{k}_\perp^2}{2k^+} \quad (2.15)$$

and CT is an energy (p^-) independent counter-term. The determination of this counter-term, such that one obtains a complete equivalence with the Schwinger Dyson approach, is in fact the main achievement of this paper. First we want to make sure that the counter-term renders the self-energy finite. This can be achieved by performing a "zero-energy subtraction" with a free propagator, analogous to adding self-induced inertias to a LF Hamiltonian, yielding

$$CT = g^2 \int_0^{p^+} dk^+ \int \frac{d^2 k_\perp}{16\pi^3} \frac{\frac{\lambda^2 + \vec{k}_\perp^2}{2k^+}}{k^+ (p^+ - k^+) D_0^{LF}} + \frac{\Delta m_{ZM}^2}{2p^+}, \quad (2.16)$$

where

$$D_0^{LF} = -\frac{M_0^2 + (\vec{p} - \vec{k})_\perp^2}{2(p^+ - k^+)} - \frac{\lambda^2 + \vec{k}_\perp^2}{2k^+} \quad (2.17)$$

and where we denoted the finite piece by Δm_{ZM}^2 (for *zero-mode*), since we suspect that it arises from the dynamics of the zero-modes. M_0^2 is an arbitrary scale parameter. We will construct the finite piece (Δm_{ZM}^2) so that there is no dependence on M_0^2 left in CT in the end.

At this point, only the infinite part of CT is unique [12], since it is needed to cancel the infinity in the k^+ integral in Eq. (2.14), while the finite (w.r.t. the k^+

³Note that in a LF formulation, G_+ and G_- are not necessarily the same.

integral) piece (i.e. Δm_{ZM}^2) seems arbitrary.⁴ Below we will show that it is not arbitrary and only a specific choice for Δm_{ZM}^2 leads to agreement between the SD and the LF approach.

Note that the equation for the self-energy can also be written in the form

$$\begin{aligned}\Sigma_+^{LF}(p) &= g^2 \int_0^{p^+} \frac{dk^+}{k^+} \int \frac{d^2 k_\perp}{8\pi^3} p_F^- G_+ (2p_F^+ p_F^-, \vec{p}_{\perp F}^2) + CT \\ \Sigma_-^{LF}(p) &= g^2 \int_0^{p^+} \frac{dk^+}{k^+} \int \frac{d^2 k_\perp}{8\pi^3} p_F^+ G_- (2p_F^+ p_F^-, \vec{p}_{\perp F}^2) \\ \Sigma_0^{LF}(p) &= -g^2 \int_0^{p^+} \frac{dk^+}{k^+} \int \frac{d^2 k_\perp}{8\pi^3} G_0 (2p_F^+ p_F^-, \vec{p}_{\perp F}^2),\end{aligned}\quad (2.18)$$

where

$$\begin{aligned}p_F^+ &\equiv p^+ - k^+ \\ p_F^- &\equiv p^- - \frac{\lambda^2 + \vec{k}_\perp^2}{2k^+} \\ \vec{p}_{\perp F} &\equiv \vec{p}_\perp - \vec{k}_\perp\end{aligned}\quad (2.19)$$

One can prove this by simply comparing expressions! Bypassing the use of the spectral function greatly simplifies the numerical determination of the Green's function in a self-consistent procedure.

C. DLCQ solution

There are reasons why one might be sceptical about the Green's function approach to the LF formulation of the model: First we used a four-component formulation which resembles a covariant calculation. Furthermore, we introduced spectral representations for the Green's functions and assumed certain properties [Eq.(2.9)]. Since we were initially also sceptical, we performed the following calculation: First we formulated the above model as a Hamiltonian DLCQ problem [13] with anti-periodic boundary conditions for the fermions and periodic boundary conditions for the bosons in the longitudinal direction. Zero modes ($k^+ = 0$) were omitted. This is a standard procedure and we will not give any details here. The only nontrivial step was the choice of the kinetic energy for the fermion, which we took, using Eq. (2.16), to be

$$T = \sum_{\vec{p}_\perp} \sum_{p^+=1,3,\dots}^\infty T(p) (b_p^\dagger b_p + d_p^\dagger d_p), \quad (2.20)$$

with

⁴Note that what we called the "finite piece" w.r.t. the k^+ integral is still divergent when one integrates over $d^2 k_\perp$ without a cutoff!

$$\begin{aligned}T(p) &= \frac{m^2 + \vec{p}_\perp^2 + \Delta m_{ZM}^2}{p^+} \\ &+ \sum_{\vec{q}_\perp} \sum_{q^+=1,3,\dots}^{p^+-2} \frac{1}{q^{+2}(p^+ - q^+)} \frac{m^2 + \vec{q}_\perp^2}{\frac{\lambda^2 + (\vec{p}_\perp - \vec{q}_\perp)^2}{k^+ - q^+} + \frac{m^2 + \vec{q}_\perp^2}{q^+}}\end{aligned}\quad (2.21)$$

(some cutoff, such as a sharp momentum cutoff, is implicitly assumed). Having obtained the eigenvalues of the DLCQ Hamiltonian, we then determined the Green's function self-consistently, by iteratively solving Eq. (2.18), using the same cutoffs as in the DLCQ calculation: the same transverse momentum cutoff and discrete k^+ summations instead of the integrals. The result was that the invariant mass at the first pole of the self-consistently determined Green's function coincides to at least 10 significant digits (!) with the invariant mass of the physical fermion as determined from the DLCQ diagonalization. This result was independent of the cutoff — as long as the same cutoff was used in both the Green's function and the DLCQ approach. This proves that the self-consistent Green's function calculation and the DLCQ calculation are in fact completely equivalent for our toy model. This is a very useful result, since it allows us to formally perform the continuum limit (replace sums by integrals) — a step that is clearly impossible for a DLCQ calculation.

D. Comparing the LF and SD solutions

Having established the equivalence between the Green's function method and the DLCQ approach, we can now proceed to compare the Green's function approach (in the continuum) with the Schwinger-Dyson approach. Motivated by considerations in Ref. [1], we make the following ansatz for ZM:

$$\Delta m_{ZM}^2 = g^2 \int_0^\infty dM^2 \int \frac{d^2 k_\perp}{8\pi^3} \rho_+^{LF}(M^2, \vec{p}_{F\perp}^2) \ln \frac{M^2}{M_0^2 + \vec{p}_{F\perp}^2}. \quad (2.22)$$

The motivation for this particular ansatz becomes obvious one we rewrite the expression for Σ_+^{LF} : For this purpose, we first note that

$$\begin{aligned}&\frac{p^- - \frac{\lambda^2 + \vec{k}_\perp^2}{2k^+}}{k^+(p^+ - k^+)D^{LF}} + \frac{\frac{\lambda^2 + \vec{k}_\perp^2}{2k^+}}{k^+(p^+ - k^+)D_0^{LF}} \\ &= \frac{p^- - \frac{p^+ - k^+}{p^+}}{k^+(p^+ - k^+)D^{LF}} - \frac{1}{p^+} \frac{\partial}{\partial k^+} \ln \left[\frac{D^{LF}}{D_0^{LF}} \right].\end{aligned}\quad (2.23)$$

Together with the normalization condition

$\int_0^\infty dM^2 \rho_+^{LF}(M^2, \vec{k}_\perp^2) = 1$, this implies

$$\Sigma_+^{LF}(p) = g^2 \frac{p^-}{p^+} \int_0^\infty dM^2 \int_0^{p^+} dk^+ \int \frac{d^2 k_\perp}{16\pi^3} \frac{(p^+ - k^+) \rho_+^{LF}(M^2; (\vec{p} - \vec{k})_\perp^2)}{k^+(p^+ - k^+)D^{LF}}$$

$$\begin{aligned}
& -\frac{g^2}{2p^+} \int_0^\infty dM^2 \int \frac{d^2 k_\perp}{8\pi^3} \rho_+^{LF}(M^2, \vec{p}_{F\perp}^2) \ln \frac{M^2}{M_0^2 + \vec{p}_{\perp F}^2} \\
& + \frac{\Delta m_{ZM}^2}{2p^+} \\
& = g^2 \frac{p^-}{p^+} \int_0^\infty dM^2 \int_0^{p^+} dk^+ \int \frac{d^2 k_\perp}{16\pi^3} \frac{(p^+ - k^+) \rho_+^{LF}(M^2, (\vec{p} - \vec{k})_\perp^2)}{k^+(p^+ - k^+) D^{LF}},
\end{aligned} \tag{2.24}$$

where we used our particular ansatz for Δm_{ZM}^2 [Eq. (2.22)]. Thus, for our particular choice for the finite piece of the kinetic energy counter term, the expression for Σ_+^{LF} and Σ_-^{LF} are almost the same — the only difference being the replacement of ρ_+^{LF} with ρ_-^{LF} and an overall factor of p^-/p^+ . Furthermore, the most important result of this paper is a direct comparison (take $x = k^+/p^+$) shows that the same spectral densities that provide a self-consistent solution to the SD equations (2.5) also yield a self-consistent solution to the LF equations, provided one chooses

$$\begin{aligned}
\rho_+^{LF}(M^2, \vec{k}_\perp^2) &= \rho_-^{LF}(M^2, \vec{k}_\perp^2) = \rho_L(M^2, \vec{k}_\perp^2) \\
\rho_0^{LF}(M^2, \vec{k}_\perp^2) &= \rho_0(M^2, \vec{k}_\perp^2).
\end{aligned} \tag{2.25}$$

In particular, the physical masses of all states (in the sector with fermion number one) must be the same in the SD and the LF framework.

In the formal considerations above, we found it convenient to express Δm_{ZM}^2 in terms of the spectral density. However, this is not really necessary since one can express it directly in terms of the Green's function

$$\Delta m_{ZM}^2 = g^2 p^+ \int_{-\infty}^0 dp^- \int \frac{d^2 p_\perp}{4\pi^3} \left[G_+(2p^+ p^-, \vec{p}_\perp^2) - \frac{1}{2p^+ p^- - \vec{p}_\perp^2 - M_0^2} \right]. \tag{2.26}$$

Analogously, one can also perform a "zero-energy subtraction" in Eq. (2.18) with the full Green's function, i.e. by choosing

$$CT = -g^2 \int_0^{p^+} dk^+ \int \frac{d^2 k_\perp}{8\pi^3} \tilde{p}_F^- G_+(2p_F^+ \tilde{p}_F^-, \vec{p}_{\perp F}^2), \tag{2.27}$$

with $\tilde{p}_F^- = -(\lambda^2 + \vec{k}_\perp^2)/2k^+$. This expression turns out to be very useful when constructing the self-consistent Green's function solution. We used both ansätze [Eqs. (2.26) and (2.27)] to determine the physical masses of the dressed fermion. In both cases, numerical agreement with the solution to the Euclidean SD equations was obtained.

Note that, in a canonical LF calculation (e.g. using DLCQ) one should avoid expressions involving G_+ , since it is the propagator for the unphysical ("bad") component of the fermion field that gets eliminated by solving the constraint equation. However, since the model that

we considered has an underlying Lagrangian which is parity invariant, one can use $G_+ = G_-$ for the self-consistent solution and still use Eq. (2.26) or Eq. (2.27) but with G_+ replaced by G_- .

III. SUMMARY

We studied a simple 3+1 dimensional model with "QCD-inspired" degrees of freedom which exhibits spontaneous breakdown of chiral symmetry. The methods that we used were the Schwinger-Dyson approach, a LF Green's function approach and DLCQ. The LF Green's function approach was used to "bridge" between the SD and DLCQ formulations in the following sense: On the one hand, we showed analytically that the LF Green's function solution to the model is equivalent to the SD approach. On the other hand we verified numerically that by discretizing the momentum integrals, that appeared in the LF Green's function approach, agreement between the LF Green's function approach and DLCQ. Hence we have shown that the SD solution and the DLCQ solution are equivalent. This remarkable result implies that even though the LF calculation was done without explicit zero-mode degrees of freedom, its solution contain the same physics as the solution to the SD equation — including dynamical mass generation for the fermions.

However, we have also shown that the equivalence between the LF approaches and the SD approach only happens with a very particular choice for the fermion kinetic mass counter-term in the light-front framework. Our calculation also showed that the current quark mass of the SD calculation is to be identified with the "vertex mass" in the LF calculation — provided the same cutoffs are being used in both calculations. This result makes sense, considering that both the current quark mass and the LF vertex mass are the only parameters that break chiral symmetry explicitly. The mass generation for the fermion in the chiral limit of the LF calculation occurs through the kinetic mass counter-term (which does not break chiral symmetry) [10].⁵ Our results contradict Ref. [14], where it has been *ad hoc* suggested that the renormalized vertex mass remains finite in the chiral limit to account for spontaneous breaking of chiral symmetry.

Our work presents an explicit 3+1 dimensional example that there is no conflict between chiral symmetry breaking and trivial LF vacua provided the renormalization is properly done. In our formal considerations, we related the crucial finite piece of the kinetic mass counter-term to the spectral density. Several alternative determinations (which might be more suitable for a

⁵We should add that the "kinetic mass counter-term" did depend on the transverse momentum of the fermion for most cutoffs other than a transverse momentum on the fermion.

practical calculation) are conceivable: parity invariance for physical observables [9], more input (renormalization conditions) such as fitting the fermion or “pion” mass.

However, one must be careful with this result in the following sense: although we have provided an explicit example which shows that, even in a 3+1 dimensional model with χSB for $m \rightarrow 0$, LF Hamiltonians without explicit zero-modes can give the right physics, we are still far from understanding whether this is possible in full QCD and how complicated the effective LF Hamiltonian for full QCD needs to be. More work is necessary to answer these questions.

As an extension of this work we had planned to study the pion in the chiral limit of a 1+1 dimensional version of this model using the LF framework.⁶ We were not able to derive an analog of the Green’s function equations for the pion, so we had to resort to a brute force DLCQ calculation. Numerical convergence, which was acceptable for the fermion, was very poor for the pion in the chiral limit, and we were thus not able to demonstrate that it emerges naturally as a massless particle. Nevertheless, we expect that other numerical techniques, which treat the end point behavior of the LF wavefunctions more carefully than DLCQ, should yield a massless pion for this model.

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⁶Even in 1+1 dimensions, one expects a massless boson in the chiral limit because of $N_C \rightarrow \infty$. One can show this in the SD formalism since the solution for the self-energy equation for the fermion also solves the Bethe-Salpeter equation for the pseudoscalar bound state with zero mass.